EXPLOITING THE CYCLOSTATIONARITY OF RADAR CHIRP SIGNALS WITH TIME-VARYING FILTERS

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ABSTRACT

A time-varying filter is proposed which improves by 5 dB upon traditional FRESH and Wiener filters when rejecting a pulsed radar signal. The filter is a Time-Varying FRESH (TV-FRESH) filter, which applies different sets of filter weights in a periodic manner, with the same periodicities of the received signal. Matching the periodicities of the filter to that of the signal improves the rejection of interference, producing a better estimate of the desired signal. The simulated results show mitigating the interference from a radar signal to an Orthogonal Frequency Division Multiplexing (OFDM) signal.

***Index Terms*—** cyclostationary, radar, TV-FRESH, optimal filter

1. INTRODUCTION

This paper proposes a novel filtering structure which is able to exploit the cyclostationarity of signals with time-varying statistics. The motivating example is a pulsed radar signal interfering with an OFDM signal, a situation which will become more common due to spectrum sharing [1]. The filter proposed in this paper shows a 5 dB improvement over both the FRESH filter and the Wiener filter.

The proposed filter is a time-vary FRESH (TV-FRESH) filter, which exploits the cyclostationarity of the received signal and applies different sets of filter weights in a periodic nature, similar to a polyphase filter bank. The periodicities of the radar signaling, including its chirp rate and its Pulse Repetition Frequency (PRF), are incorporated into the filter, a capability unique to the TV-FRESH filter.

The novelty of the paper is described in the following list:

- The TV-FRESH filter creates the ability to exploit time-varying cyclostationarity
- The filter structure applies weights in a periodic nature, improving upon existing FRESH filters
- Gives a 5 dB gain in simulated results over traditional filters
- Provides a physical layer solution to the OFDM and radar co-existence problem
- Does not require coordination between the OFDM and radar transmitters

The exploitation of cyclostationary signals with time-varying filters has been investigated prior. Almost Cyclostationary Signals (ACS) were described in [2] and the FRESH filtering of ACS signals is described by [3], where the cycle frequencies were no longer required to be harmonically related. Incorporation of conjugate-linear filtering [4] allowed conjugate spectral redundancy to be exploited by the FRESH filter [5]. Frequency domain and adaptive FRESH filters were later developed. The frequency domain FRESH filter was first described by [6] and adaptive versions in [7]. More recently FRESH filters have been used for the equalization of doubly-selective channels [8] and robust communications [9].

The remainder of the paper is outlined as follows. Background on cyclostationary signals and FRESH filtering is given in Section 2. The TV-FRESH filter is proposed in in Section 4. Simulated results are presented in Section 5 and the paper is concluded in Section 6.

2. BACKGROUND

In this section background material is given on cyclostationary signals and FRESH filtering.

2.1. Cyclostationary Signals

The cyclic autocorrelation function (CAF) is used to determine second-order periodicity of frequency α is present within $x(t)$ [5],

$$R_x^\alpha(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x\left(t+\frac{\tau}{2}\right) x^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi\alpha t} dt. \quad (1)$$

The cyclic autocorrelation is also component of a generalized Fourier Series [3] of the autocorrelation function,

$$R_x(t,\tau) = \sum_\alpha R_x^\alpha(\tau) e^{j2\pi\alpha t}, \quad (2)$$

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and related through:

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} R_x(t,\tau) e^{-j2\pi\alpha t} dt. \tag{3}$$

Wide sense stationary signals contain no second-order periodicities, meaning the autocorrelation function does not contain any additive sine wave components of frequency $\alpha \neq 0$. Therefore their autocorrelation function can be written as [10],

$$R_x(t,\tau) = R_{t,x}^{0}(\tau). \tag{4}$$

Wide sense cyclostationary signals contain second-order periodicity, and as such the autocorrelation function is periodic,

$$R_x(t,\tau) = \sum_{n=-\infty}^{\infty} R_x^{\alpha_n}(\tau) e^{j2\pi\alpha_n t}, \tag{5}$$

where the cycle frequencies $\{\alpha_n\}$ are all commensurate [10] with period T,

$$\alpha_n = \frac{n}{T},\ n = 0, \pm1, \pm2, \ldots\ . \tag{6}$$

This can be interpreted as a Fourier series, with coefficients $R_x^{\frac{n}{T}}(\tau)$ and frequencies $\frac{n}{T}$ [11],

$$R_x(t,\tau) = \sum_{n=-\infty}^{\infty} R_x^{\frac{n}{T}}(\tau) e^{j2\pi\frac{n}{T}t}. \tag{7}$$

The cyclostationarity of many analog and digital single carrier signals are described in [12, 13]. A signal is almost cyclostationary when the autocorrelation function is an almost periodic function,

$$R_x(t,\tau) = \sum_{n\in A} R_x^{\alpha_n}(\tau) e^{j2\pi\alpha_n t}. \tag{8}$$

where the cycle frequencies $A = \{\alpha_n\}$ may not be commensurate [10].

2.2. FRESH Filtering

The ability to exploit the spectral redundancy and conjugate spectral redundancy of ACS signals is described in [5]. The FRESH filter is described by:

$$\begin{aligned}\hat{d}(t) = &\sum_{m=0}^{M-1} a_m(t) \otimes x(t) e^{j2\pi\alpha_m t} \\ &+ \sum_{n=0}^{N-1} b_n(t) \otimes x^*(t) e^{j2\pi\beta_n t}.\end{aligned} \tag{9}$$

The FRESH filter incorporates the periodicities α and β in the signal $x(t)$ to better improve the estimate of the desired signal, making use of spectrally redundant information.

3. SIGNAL MODEL

A chirp signal is a complex exponential whose frequency is swept over time with some period T_c. The model of the periodic chirp signal is given by [14]:

$$c(t) = \sum_{m=-\infty}^{\infty} e^{j2\pi f_c (t-mT_c)^2} q(t - mT_c), \tag{10}$$

where f_c is the rate of change in frequency, or the chirp rate, T_c is the period of the chirp, and $q(t)$ is a rectangular pulse of length T_c. The change in frequency creates a form of time-varying diversity which can be exploited with an appropriate filter.

The sampled chirp signal is a discrete-time almost cyclostationary signal, [14], and as such the discrete time chirp $c(kT_s)$ can be represented by the generalized Fourier Series [15],

$$c(kT_s) = \sum_{\alpha} c_{\alpha} e^{j2\pi\alpha kT_s} \tag{11}$$

with discrete time cycle frequencies and conjugate cycle frequencies,

$$\alpha = \frac{n}{T_s T_c},\ n = 0, \pm1, \pm2,\ \ldots\ , \frac{T_s T_c}{2}, \tag{12}$$

$$\beta = \frac{p}{T_s T_c},\ p = 0, \pm1, \pm2,\ \ldots\ , \frac{T_s T_c}{2}. \tag{13}$$

The chirp signal $d(kT_s)$ has many cycle frequencies at all of the harmonics of its chirp period T_c.

The periodicities of the OFDM signal are known to be $\alpha = \frac{n}{N_T}$, for $n = 0, \pm1, \pm2,\ \ldots\ , \pm\frac{N_T}{2}$ where N_T is the length of the OFDM symbol in number of samples, including the cyclic prefix [16].

4. TV-FRESH FILTER

The TV-FRESH filter uses a tapped delay line of length B in order to exploit the periodic nature of the radar signal, and the length B will be designed to match the periodicities of the radar signal. The TV-FRESH is a frequency domain filter, with the input signal being segmented into B OFDM symbols, $X_{l,0}(f), X_{l,1}(f),\ \ldots\ , X_{l,B-1}(f)$, where l describes the sets of frequency domain samples over time. The filter then estimates the c^{th} set of B frequency domain samples [9],

$$\begin{aligned}\hat{D}_{l,c}(f) = \sum_{b=0}^{B-1} \Bigg[&\sum_{u=0}^{U_{c,b}-1} G_{c,b,u}(f) X_{l,b}(f - \alpha_{c,b,u}) \\ &+ \sum_{v=0}^{V_{c,b}-1} H_{c,b,v}(f) X_{l,b}^*(-f + \beta_{c,b,v}) \Bigg].\end{aligned} \tag{14}$$

The TV-FRESH has two sets of filters, $G_{c,b,u}(f)$ and $H_{c,b,u}(f)$, corresponding to the filters $a_m(t)$ and $b_n(t)$ from (9). The cycle frequencies $\alpha_{c,b,u}$ and $\beta_{c,b,u}$ also correspond to the periodicities α and β of (9). The periodicities $\alpha_{c,b,u}$ and $\beta_{c,b,u}$ are now time-varying also. For example, $\alpha_{1,3,k}$ represents the k^{th} periodicity between the third and first sets of frequency domain samples, $X_{l,1}(f)$ and $X_{l,3}(f)$. The values $U_{c,b}$ and $V_{c,b}$ represent the number of periodicities for $\alpha_{c,b,u}$ and $\beta_{c,b,u}$ that are included in the filter.

The MMSE weights of the TV-FRESH filter are found by forming the Mean Squared Error (MSE), taking its derivative and setting equal to zero, as in [9]. The filter error for OFDM symbol c is defined as:

$$E_{l,c}(f) = D_{l,c}(f) - \hat{D}_{l,c}(f), \tag{15}$$

The MSE is minimized by taking the derivative with respect to the two sets of filters and setting equal to zero,

$$\frac{\partial}{\partial G^*_{c,p,k}(f)} \mathbb{E}\left\{E_{l,c}(f)E^*_{l,c}(f)\right\} = 0, \tag{16}$$

$$\frac{\partial}{\partial H^*_{c,m,n}(f)} \mathbb{E}\left\{E_{l,c}(f)E^*_{l,c}(f)\right\} = 0. \tag{17}$$

Substituting the conjugated filter error (15) into the partial derivatives results in two orthogonal projections,

$$\mathbb{E}\left\{E_{l,c}(f)X^*_{l,p}\left(f - \alpha_{c,p,k}\right)\right\} = 0, \tag{18}$$

$$\mathbb{E}\left\{E_{l,c}(f)X_{l,m}\left(-f + \beta_{c,m,n}\right)\right\} = 0. \tag{19}$$

Substituting the filter error (15) into the projections (18) and (19) gives the filter weight design equations,

$$\begin{aligned} S^{\alpha_{c,p,k}}_{d_c,x_p}\left(f - \frac{\alpha_{c,p,k}}{2}\right) = \sum_{b=0}^{B-1}\Bigg[&\sum_{u=0}^{U_{c,b}-1} G_{c,b,u}(f) S^{\alpha_{c,p,k}-\alpha_{c,b,u}}_{x_c,x_b}\left(f - \frac{\alpha_{c,p,k}+\alpha_{c,b,u}}{2}\right) \\ &+ \sum_{v=0}^{V_{c,b}-1} H_{c,b,v}(f)\, S^{\beta_{c,b,v}-\alpha_{c,p,k}}_{x_c,x^*_b}\left(f - \frac{\beta_{c,b,v}+\alpha_{c,p,k}}{2}\right)^*\Bigg] \end{aligned} \tag{20}$$

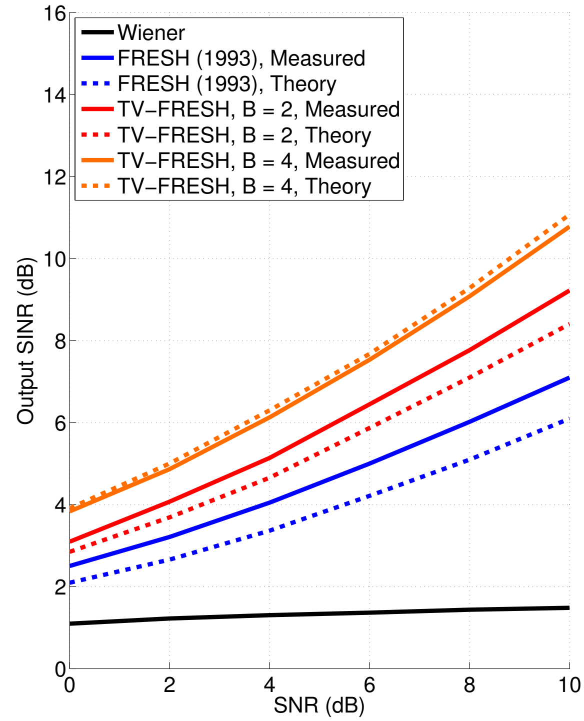
$$p = 0, 1, \ldots, B-1; \quad k = 0, 1, \ldots, U_{c,p} - 1,$$

$$\begin{aligned} S^{\beta_{c,m,n}}_{d_c,x^*_m}\left(f - \frac{\beta_{c,m,n}}{2}\right) = \sum_{b=0}^{B-1}\Bigg[&\sum_{u=0}^{U_{c,b}-1} G_{c,b,u}(f) S^{\beta_{c,m,n}-\alpha_{c,b,u}}_{x_c,x^*_b}\left(f - \frac{\beta_{c,m,n}+\alpha_{c,b,u}}{2}\right) \\ &+ \sum_{v=0}^{V_{c,b}-1} H_{c,b,v}(f)\, S^{\beta_{c,m,n}-\beta_{c,b,v}}_{x_c,x_b}\left(-f + \frac{\beta_{c,m,n}+\beta_{c,b,v}}{2}\right)\Bigg] \end{aligned} \tag{21}$$

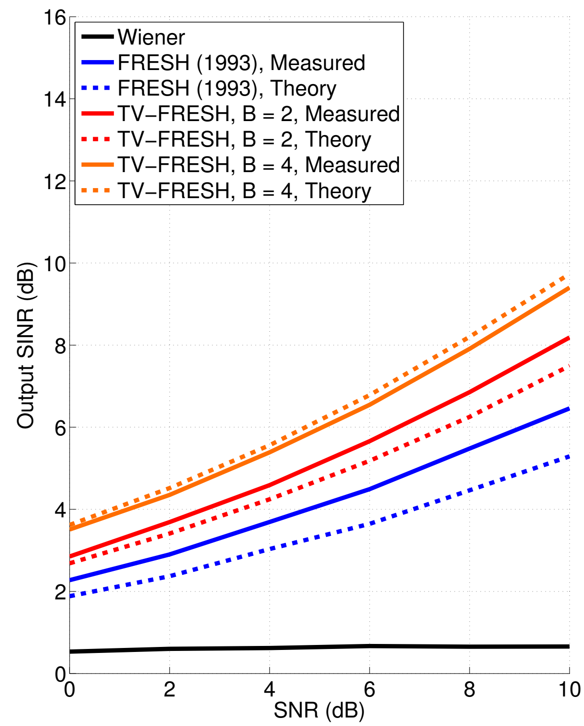
$$m = 0, 1, \ldots, B-1; \quad n = 0, 1, \ldots, V_{c,m} - 1.$$

The theoretical SINR at the output of the filter (14) is given by:

$$\lambda_l = \frac{\mathbb{E}\left\{\left|D_{l,c}(f)\right|^2\right\}}{\mathbb{E}\left\{\left|D_{l,c}(f) - \hat{D}_{l,c}(f)\right|^2\right\}}. \tag{22}$$



(a) The output SINR with an AWGN channel.



(b) The output SINR with a Rayleigh fading channel.

Fig. 1: The SINR comparison for the filtering methods.

5. SIMULATION RESULTS

The interference scenario is an OFDM signal being interfered with by a radar chirp signal. The first case only considers an Additive White Gaussian Noise (AWGN) channel, while the second is a Rayleigh frequency selective fading channel with weights $[-1.21-1.37j, 0.58-0.61j, 0.06+0.06j]$ on the OFDM signal and $[0.72-0.86j, 0.09-0.63j, -0.18-0.53j]$ on the interference. The OFDM signal uses 16 subcarriers with a cyclic prefix length of 8 samples. The chirp is 128 samples long, and the radar pulse period is 512 samples. The SIR = -10 dB.

A small number of subcarriers are used for illustrative purposes, while increasing the number of subcarriers will improve the SINR at the cost of additional complexity.

At large input SNR values, the gain of the TV-FRESH over the traditional FRESH filter is 5 dB. The TV-FRESH is able to effectively remove the radar signal perfectly at large input SNR values and with $B = 4$, with the output SINR being nearly equivalent to the input SNR value. The Rayleigh channel gives the TV-FRESH a 4.5 dB improvement over the traditional FRESH.

6. CONCLUSION

An optimal TV-FRESH filter has been proposed for sampled GACS signals. The TV-FRESH filter is able to track and exploit the cyclostationarity and time-varying statistics of the radar signal, providing a gain of 5 dB over the FRESH filter and Wiener filter. The gain comes from the ability of the TV-FRESH to apply time-varying sets of filter weights according to the periodicities in the received signal, a capability the FRESH filter and Wiener filters do not have.

Future work would include incorporating an adaptive feedback algorithm into the filtering structure, exploiting time-varying spectral redundancy in the spatial domain, and adapting the filtering structure itself as signaling parameters of a radar waveform change.

7. REFERENCES

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